

## Is cosmological constant screened in Liouville gravity with matter?

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Naka-ku Hamamatsu-shi, Shizuoka, Japan***Abstract**

There has been a proposal that infrared quantum effects of massless interacting field theories in de-Sitter space may provide time-dependent screening of the cosmological constant. As a concrete model of the proposal, we study the three loop corrections to the energy-momentum tensor of massless  $\lambda\phi^4$  theory in the background of classical Liouville gravity in  $D = 2$  dimensional de-Sitter space. We find that the cosmological constant is screened in sharp contrast to the massless  $\lambda\phi^4$  theory in  $D = 4$  dimensions due to the sign difference between the cosmological constant of the Liouville gravity and that of the Einstein gravity. We also propose an alternative possibility to renormalize the  $\lambda\phi^4$  theory so that the de-Sitter invariance is intact and the cosmological constant is never screened. The de-Sitter invariance is recovered by adding time-dependent infrared counter-terms.

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# 1 Introduction

Recent observation of dark energy in our universe has led to the conviction that there exists a tiny but positive value of the cosmological constant  $\Lambda$ . It means that our space-time is de Sitter (dS) space with the Hubble constant  $H$  being  $\sqrt{\Lambda}$ . There has been a proposal that the strong infrared (IR) divergence property of the quantum corrections on dS space may explain the smallness of  $\Lambda$  in our current universe (so called cosmological constant problem). If we are to calculate quantum corrections to the value of the cosmological constant today, we have to deal with quantum field theories on dS space. This can be performed by using the in-in formalism or Schwinger-Keldysh formalism [1, 2, 3, 4, 5, 6].

The IR divergence makes it difficult to keep the dS invariance in the propagators of massless fields. The question of whether or not to preserve the dS invariance has been addressed repeatedly [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] (for IR effects during cosmological inflation, see [20] for a review). The ambiguity in imposing the boundary condition on the propagators at the horizon has also been discussed in [21]. A complete agreement has not been reached yet in the evaluation of loop corrections to the energy-momentum (EM) tensor  $T_{\mu\nu}$  in quantum gravity coupled to matter in the four dimensions (4D) [22, 23, 24, 25]. In this situation we believe that studying quantum gravity and matter loop effects on the EM tensor  $T_{\mu\nu}$  in two-dimensional (2D) dS space may help clarify the problem of quantum corrections to the cosmological constant  $\Lambda$ .

Our  $D = 2$  dimensional model for quantum gravity coupled to matter fields is based on the 2D Liouville field theory [26] minimally coupled to matter fields. The Liouville field is a Weyl factor of the metric and originally it has no kinetic term at the classical level. The origin of the kinetic term is from the Weyl non-invariant measure of the path integral for quantum gravity such as Weyl anomaly. The resulting Liouville field theory captures the non-perturbative dynamics of the low energy effective field theory of the 2D quantum gravity [27, 28] and hence contains the complete information of the quantum gravity as an ordinary quantum field theory. Once we derive the Liouville field theory as the 2D quantum gravity (Liouville gravity), one may take the classical limit by assuming the large number of matter fields. The Liouville field theory is conformally invariant, and at least classically, there is no subtleties in the dS background. The “coupling constant” of the potential term in the Liouville field theory is related to the cosmological constant of 2D quantum gravity and renormalized by the matter loop effects. Similarly to the higher-dimensional Einstein equation with the dS breaking quantum matter EM tensor, we expect that the subtle quantum IR effects of the interacting massless fields may significantly affect the dynamics of the Liouville gravity.

Constructing a 2D model based on the Liouville field theory is also motivated by (and

is related to) an old work by Polyakov for the IR screening of the cosmological constant [29]. The Weyl factor of the metric plays a leading role there. There are attempts to the screening mechanism from the dynamics of the Weyl factor in 4D gravity [30, 31, 32]. If such a mechanism is really at work, it would significantly affect our mind-set to understand the cosmological constant problem. Since the Liouville gravity coupled to quantum matter is a power-counting renormalizable field theory, we should be able to answer the question unambiguously.<sup>1</sup> To be screened or not to be screened, that is the question.

In this work we restrict ourselves to the perturbative effects from the matter sector. As a concrete example, we choose a scalar field theory with  $\lambda\phi^4$  interaction on dS background. Evaluation of the matter loop corrections to  $T_{\mu\nu}$  to higher loops is carried out by using the propagator for a massless scalar field  $\phi$  with a dS symmetry breaking term. Hence the EM tensor acquires the logarithmic time dependence which is often referred to as the IR logarithm,  $\ln a(t)$ . We find that our result shows the screening effect of the effective cosmological constant up to order  $\lambda^3$  corrections. We also find that the degree of IR divergence from massless scalar fields in 2D dS space is the same as that in 4D at least within a perturbative computation.

This conclusion however is puzzling at least for one reason. In flat Minkowski space, the IR limit of  $\lambda\phi^4$  theory in  $D = 2$  dimensions is equivalent to a free massless Majorana fermion (or critical Ising model) from the Landau-Ginzburg construction [33, 34]. The free Majorana fermion is conformally invariant and does not show any IR pathology in dS space. The cosmological constant induced by the free Majorana fermion is never screened. While the possibility that the strongly coupled fixed point with the fine-tuning of the mass parameter may alter the scenario remains, we will offer an alternative way to renormalize the  $\lambda\phi^4$  theory so that the dS invariance is intact at the sacrifice of the naive equations of motion. In this picture, the connection to the free massless Majorana fermion is more transparent.

Actually, all the IR pathologies we find are given by the local term in the effective action, and we can always remove them by adding the dS breaking counter-terms. Very roughly speaking, one can add the time-dependent “classical cosmological constant term” to cancel the “quantum” screened cosmological constant term. We find a similar but more simplified situation in the puzzle of (in)equivalence between the Sine-Gordon model and massive Thirring model in dS background proposed the literature [35]. We find that by adding non-conventional dS breaking local counter-terms (with which the quantum dS breaking is cancelled in the final effective action), we can resolve the puzzle there. We will report the details in a separate publication. We expect that the argument is universally

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<sup>1</sup>In [25], a possibility to screen the cosmological constant in non-unitary time-like Liouville theory was discussed. See also footnote 5 in comparison with their approach.

applicable.

Whether such counter-terms are allowed or should be added must be determined from some other principles. If we stick to the dS invariance, there is no reason not to add them unless it is inconsistent with more important principles such as gauge invariance. The mechanism should work in other space-time dimensions while at this stage, we are not certain if the obstruction to recover the dS invariance from IR counter-terms existed for gravitons or gauge fields in higher dimensions.

In the next section, we briefly review the IR divergence originated from non-conformally invariant massless scalar fields in dS space in general space-time dimensions. Introduction of an IR cutoff for momentum integration leads to the IR logarithm  $\ln a$  in the coordinate space propagator which immediately breaks a part of dS symmetry, namely dilatation invariance,  $\eta \rightarrow b\eta$ ,  $x^i \rightarrow bx^i$ . It makes the cosmological constant time dependent through the Einstein equation. We introduce our 2D model of Liouville gravity and matter loop corrections in section 3. In section 4, we compute the perturbative corrections of order  $\lambda^2$  to the EM tensor from massless scalar loops in  $\lambda\phi^4$  theory. In section 5, we discuss the possibility of the dS non-invariant counter-terms designed for cancelling the dS breaking IR logarithms. We conclude with discussion in section 6. In appendix A, we report the detailed computation of the order  $\lambda^2$  corrections to the EM tensor in  $\lambda\phi^4$  theory.

## 2 Cosmological constant problem and infrared effects

### 2.1 Infrared divergences in de-Sitter space

In this paper, we work on quantum field theories on  $D$ -dimensional dS space. Among various choices of coordinates, we mainly use the Poincaré coordinate, in which it is manifested that dS space is conformally flat. The dS geometry is expressed by the metric

$$ds^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x}, \quad (2.1)$$

where the scale factor  $a$  is given by Hubble constant  $H$ , and the conformal time  $\eta$  as

$$a = e^{Ht} = -\frac{1}{H\eta}, \quad H(t) \equiv \frac{\dot{a}(t)}{a(t)}. \quad (2.2)$$

Here  $\eta$  is related to the physical time  $t$  by

$$\eta = -\frac{1}{H}e^{-Ht}, \quad (2.3)$$

and it runs from  $-\infty$  to 0 ( $-\infty \leq t \leq \infty$ ). By using the conformal time  $\eta$ , the dS metric becomes

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x} \cdot d\vec{x}). \quad (2.4)$$

This coordinate covers the half of the global dS space.

In dS space, IR divergence property of (non-conformally invariant) massless fields is different from that in Minkowski space because large distance is affected by the dS curvature. One can easily see such a property by considering the vacuum loop graphs of massless scalar fields. They are obtained by integrating over the loop momentum  $P$  where  $P$  is physical momentum. Let us follow the explanation given in [36]. It is convenient to divide the integration region into two, UV region (sub-horizon)  $|P| > H$  and IR region (super-horizon)  $|P| < H$ . For example in 4D space-time<sup>2</sup>

$$\int d^3 P = \int_{|P|>H} d^3 P + \int_{|P|<H} d^3 P. \quad (2.5)$$

The mode function of massless minimally coupled scalar field in the Bunch-Davies vacuum [37] is given by

$$\phi_k^{D=4}(\eta) = \frac{H\eta}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}, \quad (2.6)$$

where  $k$  is a comoving momentum which is related to physical momentum  $P$  as  $P = k/a(\eta) = -kH\eta$ . In dS space the fluctuations of the massless fields (scalars and gravitons) have the scale free spectrum which behaves as  $1/P^3$  at super-horizon scale due to the second term in (2.6). The loop integral  $\int_{|P|<H} d^3 P P^{-3}$  then gives rise to a logarithmic divergent contribution at  $P \rightarrow 0$ .

Let us repeat the analysis in  $D = 2$ , which is our main focus of this paper. The mode function of the massless scalar field is given by

$$\phi_k^{D=2}(\eta) = \frac{-i}{\sqrt{2k}} e^{-ik\eta}. \quad (2.7)$$

This is completely the same form of that in Minkowski space because minimally coupled free massless scalar fields in 2D space-time are conformally invariant and we have a conformal vacuum as a dS invariant vacuum. The vacuum loop is in this case given by

$$\int_0^\infty dP \frac{1}{2P} = \int_H^\infty dP \frac{1}{2P} + \int_0^H dP \frac{1}{2P}, \quad (2.8)$$

where we again make use of physical momentum  $P = k/a(\eta)$  and divide the integral into  $P < H$  and  $P > H$  as in  $D = 4$  case even though there is no distinction between the behavior of sub-horizon and super-horizon modes. The divergence structure is the same as in  $D = 2$  Minkowski space.

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<sup>2</sup>In terms of comoving momentum, we have  $\int d^3 k = \int_{|k|>aH} d^3 k + \int_{|k|<aH} d^3 k$ . The IR cutoff is given by  $k_0 = a_i H$  [21] where  $a_i$  is the scale factor at the initial time.

As we have seen in above two examples, the origin of the IR divergence lies at (i) zero comoving momentum  $k \rightarrow 0$  or (ii) infinite future  $\eta \rightarrow 0$ . Here we regularize the IR divergence from (i) by truncating the Hilbert space at some comoving momentum  $k_0$  as an IR cutoff. As a result of this prescription, the second term in (2.5), and (2.8) give a factor  $\ln(a(\eta)/k_0)$ .<sup>3</sup>

There is a little subtlety in putting IR and UV cutoffs for momentum in the vacuum loop. To obtain the IR logarithm from the vacuum loop, the UV cutoff of the first integral in (2.5) has been taken implicitly to be *physical*  $P = \Lambda_{\text{UV}} = \text{const.}$ , which makes

$$\int_H^{\Lambda_{\text{UV}}} dP$$

constant. On the other hand, the IR cutoff in the second integral is taken to be *comoving*,  $k_0$ , and accordingly,  $P = k_0/a(t)$  is not a constant. It amounts to saying that UV cutoff of the theory does not change due to the cosmic expansion, on the other hand, the number of IR modes ( $P(t) < H$ ) increases with time. If we put the IR cutoff to be physical  $P = \Lambda_{\text{IR}} = \text{const.}$ , the time dependence disappears from the vacuum loop.

Even if the IR divergence is regularized once, owing to the exponential expansion of the universe, the dS space distance grows with time and at the same time the physical wavelengths are all red shifted. Eventually as the conformal time  $\eta$  approaches 0 ( $t$  to  $\infty$ ) IR divergence of kind (ii) appears due to  $\ln(a(\eta))$  which is often referred to as the (dS breaking) IR logarithm. From the detailed study of this IR logarithm with dS breaking, we can learn the characteristic effect of the massless fields and its consequence in dS space.

The same type of IR divergence as in the case of the vacuum loop appears in the coordinate space propagator. In section 4, we will adopt the dimensional regularization for UV divergence in the loop computations while we adopt the cutoff regularization for IR divergence. The propagator of a massless minimally coupled scalar field in  $D$ -dimensional dS space is obtained in [38, 39]. By setting  $D = 2 - \omega$  with  $\omega > 0$ , the propagator is given by

$$i\Delta(x, z) = \alpha\{\gamma(y) + \beta \ln(a(\eta)a(\eta_z))\}, \quad (2.9)$$

where

$$y \equiv \frac{-(\eta - \eta_z)^2 + (\vec{x} - \vec{z})^2}{\eta\eta_z}, \quad (2.10)$$

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<sup>3</sup> For later use we note that the IR cutoff  $k_0$  is related to the cutoff for the initial time  $t_i$  [36, 21] since the largest comoving scale is given by  $l_0 = k_0^{-1}$ , and an identification of  $l_0$  leads to that of  $t_i$  through  $a(t_i)l_0 = L_i$  with  $L_i$  the initial (physical) size of the universe. We adopt  $a(t_i) = 1$  ( $t_i = 0$ ) with  $L_i = l_0 = H^{-1}$  as a reference time for the renormalization conditions in section 4 and an initial time cutoff for the vertex integrals in Appendix A.

$$\alpha = \frac{1}{4\pi} \left( \frac{H^2}{4\pi} \right)^{-\omega}, \quad \beta = \frac{\Gamma(1-\omega)}{\Gamma(1-\frac{\omega}{2})}, \quad (2.11)$$

and

$$\begin{aligned} \gamma(y) = & -\frac{\Gamma(1-\frac{\omega}{2})}{\omega} \left( \frac{y}{4} \right)^{\frac{\omega}{2}} - \frac{\Gamma(2-\frac{\omega}{2})}{1+\frac{\omega}{2}} \left( \frac{y}{4} \right)^{1+\frac{\omega}{2}} + \beta\delta \\ & + \sum_{n=1}^{\infty} \left[ \frac{\Gamma(1-\omega+n)}{n\Gamma(1-\frac{\omega}{2}+n)} - \frac{\Gamma(2-\frac{\omega}{2}+n)}{(1+\frac{\omega}{2}+n)(n+1)!} \left( \frac{y}{4} \right)^{\frac{2+\omega}{2}} \right] \left( \frac{y}{4} \right)^n + \mathcal{O}(k_0^2), \end{aligned} \quad (2.12)$$

with

$$\delta \equiv -\pi \cot \left( \pi - \frac{\omega}{2} \pi \right) + C, \quad (2.13)$$

$$C \equiv \frac{1}{2} \ln \left( \frac{H}{k_0} \right) + \psi \left( \frac{1}{2} - \frac{\omega}{2} \right) - \psi \left( 1 - \frac{\omega}{2} \right) + \psi(1-\omega) - \gamma. \quad (2.14)$$

Here an IR cutoff  $k_0$  for comoving momentum has been introduced.  $\psi(x)$  is the digamma function and  $\gamma$  the Euler-Mascheroni constant. The distance in dS space is commonly denoted by  $y$ . The propagator (2.9) has a simple structure that the first term is manifestly dS invariant because it depends only on the distance  $y$  which respects the dS symmetry in Poincaré coordinate (a dilatation,  $D-1$  dimensional spatial rotations,  $D-1$  dimensional spatial translations and  $D-1$  dimensional special conformal transformations [11, 12, 35]). On the other hand, the second term breaks the dS invariance due to the IR logarithm.

The basic formalism to calculate the correlation functions in time dependent backgrounds is called the in-in formalism. In the in-in formalism two copies of time sheets, named by  $+$  and  $-$  are prepared and the time path is then closed:  $\int_C d\eta = \int_{-\infty}^0 d\eta_+ + \int_0^{-\infty} d\eta_-$ . All vertices in the loop diagrams are assigned  $+$  or  $-$  type. The expectation value of operator(s)  $\mathcal{O}(x)$  is given by

$$\begin{aligned} \langle \Omega | \mathcal{O}(x) | \Omega \rangle &= \langle T \{ \mathcal{O}(x) e^{-i \int_C H_{\text{int}} d\eta} \} \rangle \\ &= \langle \tilde{T} \{ e^{i \int_{\eta_i}^0 H_{\text{int}-} d\eta_-} \} T \{ \mathcal{O}(x) e^{-i \int_{\eta_i}^0 H_{\text{int}+} d\eta_+} \} \rangle \end{aligned} \quad (2.15)$$

where  $T$  and  $\tilde{T}$  stand for the usual time ordering operator and the anti-time ordering operator, respectively. Here  $|\Omega\rangle$  in the first line is the vacuum of the interacting theory, and the  $\langle \mathcal{O}(x) \cdots \rangle$  is the expectation value in the free field theory that can be computed by the Wick contraction. We introduced  $\eta_i$  as an initial time and assumed  $x$  to be  $+$  type in the second equality. Depending on the types of vertices, all  $y$  have one of four types [38].

$$\begin{aligned} y_{++}(x, x') &\equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (|\eta - \eta'| - ie)^2], \\ y_{+-}(x, x') &\equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (\eta - \eta' + ie)^2], \\ y_{-+}(x, x') &\equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (\eta - \eta' - ie)^2], \\ y_{--}(x, x') &\equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (|\eta - \eta'| + ie)^2], \end{aligned} \quad (2.16)$$



where  $y_{ab}(x, x')$  stands for  $y(x_a, x'_b)$  ( $a, b = \pm$ ) with  $e$  a positive infinitesimal. By substituting each distance  $y$ , we can construct the four propagators used in the in-in formalism. We denote them by

$$\begin{aligned} i\Delta_{++}(y_{++}) &\equiv \langle T\{\phi(x_+)\phi(x'_+)\} \rangle, \\ i\Delta_{+-}(y_{+-}) &\equiv \langle \phi(x'_-)\phi(x_+) \rangle, \\ i\Delta_{-+}(y_{-+}) &\equiv \langle \phi(x_-)\phi(x'_+) \rangle, \\ i\Delta_{--}(y_{--}) &\equiv \langle \tilde{T}\{\phi(x_-)\phi(x'_-)\} \rangle. \end{aligned} \tag{2.17}$$

The short distance ( $y \rightarrow 0$ ) limit of the propagator (2.9) is regularized by  $\omega$  and is independent of the labels  $+$ ,  $-$ , as is seen

$$\lim_{x \rightarrow z} i\Delta(x, z) = \alpha\beta \left( 2 \ln(a(\eta)) + \frac{2}{\omega} + C + \gamma + \mathcal{O}(\omega) \right). \tag{2.18}$$

## 2.2 Infrared effects on the cosmological constant $\Lambda$

In space-time dimensions  $D > 2$ , the Einstein equation describes the relation between the space-time Ricci tensor  $R_{\mu\nu}$  and the EM tensor  $T_{\mu\nu}$  due to the presence of matters,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2.19}$$

where  $R$  is scalar curvature,  $\kappa = 8\pi G$  with  $G$  being Newton's constant,  $\Lambda$  is the cosmological constant. In the vacuum states where  $T_{\mu\nu}$  is proportional to the metric, we may transfer  $T_{\mu\nu}$  to the left hand side of (2.19), and the vacuum Einstein equation takes the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda_{\text{eff}}g_{\mu\nu} = 0, \tag{2.20}$$

where

$$\Lambda_{\text{eff}} = \Lambda - \frac{\kappa}{D}T_{\rho}^{\rho}. \tag{2.21}$$

The vacuum contribution of  $T_{\mu\nu}$  is now combined with  $\Lambda$  to define the effective cosmological constant.

In view of this expression we may wonder if a large value of  $T_{\mu\nu}$  may cancel the large value of  $\Lambda$  yielding a tiny value of  $\Lambda_{\text{eff}}$  that we observe today. For such cancellation, the quantum corrections to  $T_{\mu\nu}$  is essential. This idea may or may not address the cosmological constant problem because we have yet to know what the bare cosmological constant  $\Lambda$  should be (see [40] for a review).

The situations in dS space with massless interacting fields are much more complicated. As mentioned previously, the massless scalar propagator in dS space has IR divergence.

This IR divergence is regulated by the IR cutoff and renormalized. The IR cutoff, however, introduces dS invariance breaking term from the IR logarithm  $\ln a$ . Then the expectation value of  $T_{\mu\nu}$  of the massless interacting fields in dS space explicitly depends on the IR logarithm  $\ln a$  and it becomes non-dS-covariant. As a consequence the effective cosmological constant becomes time-dependent from (2.21). This time-dependent screening effects proposed in the literature may cause the drastic effects in the fine-tuning problem of the cosmological constant.

In perturbation theory, both matter loops and gravity loops may provide sources of corrections to the cosmological constant  $\Lambda$ . Quantum effects of gravity in 4D dS space have been studied extensively for a long time (see [41, 42] for reviews). Due to the difficulty of keeping the dS invariance in the massless propagators and the ambiguity in taking account of the boundary conditions at the horizon, a complete agreement has yet to be reached in the evaluation of loop effects on the  $T_{\mu\nu}$  even after extensive studies. In the semi-classical limit, or in the large number of matter fields limit, the matter loop corrections will dominate over the gravity loop corrections, so we may treat the Einstein gravity classically while replacing  $\Lambda$  by the quantum expectation value of the matter contributions in the fixed dS background. Although such a limit is purely academic in our  $D = 4$  universe, we may still learn important lessons on screening of cosmological constant from the quantum IR effects.

The goal of this paper is to calculate quantum effects in lower dimension because IR divergence in lower dimension is stronger than that in higher dimension in Minkowski space. A question is whether we observe similar enhancement of the IR effects in  $D = 2$  dS space. We also address the question if the dS invariance may be recovered from the IR counter-terms. While we demonstrate the possibility in  $D = 2$ , the same argument may apply in higher dimensions, too.

In  $D = 2$ , the classical Einstein gravity becomes trivial, and the discussion in this section must be replaced by the other model of gravity. We opt to use the Liouville gravity that is induced by the quantum fluctuation of the Weyl mode of the metric. Again in the large number of matter fields limit, one may treat the Liouville degrees of freedom classically while replacing the effective cosmological constant term from the matter contributions in the fixed Liouville background. The details will be described in the next section.

### 3 2D model for quantum gravity with matter

#### 3.1 2D Liouville theory

In  $D = 2$  dimensions, the Einstein gravity with the Einstein-Hilbert action

$$S[g_{\mu\nu}] = \int d^2x \sqrt{-g} \frac{1}{2\kappa} (R - 2\Lambda), \quad (3.1)$$

has no dynamical degrees of freedom because the Einstein-Hilbert term is topological due to Gauss-Bonnet theorem. However, at the quantum level, the Weyl mode  $\Phi$  of the metric  $g_{\mu\nu} = e^{2\Phi} \hat{g}_{\mu\nu}$  becomes dynamical and the *quantum* gravity in  $D = 2$  dimensions is described by the dynamical Liouville field theory. Here  $\hat{g}_{\mu\nu}$  is the fiducial metric that we can choose arbitrarily. Because of this arbitrariness, the Weyl invariance (i.e.  $\hat{g}_{\mu\nu} \rightarrow e^{2\sigma} \hat{g}_{\mu\nu}$  and  $\Phi \rightarrow \Phi - \sigma$ ) of the Liouville gravity is automatically guaranteed.<sup>4</sup>

We briefly recapitulate the Liouville theory. We begin with 2D gravity field (metric)  $g_{\mu\nu}$  coupled to “matter fields” collectively called  $X$ . The action and the partition function are given by

$$S_{2D}[g_{\mu\nu}, X] = \int d^2x \sqrt{-g} \left( \frac{1}{2\kappa} (R - 2\Lambda) \right) + S_{\text{matter}}[g_{\mu\nu}, X], \quad (3.2)$$

$$Z = \int DX Dg_{\mu\nu} e^{iS_{2D}[g_{\mu\nu}, X]}. \quad (3.3)$$

In  $D = 2$ , we may (locally) parametrize the gravity fluctuation by the Liouville degrees of freedom  $g_{\mu\nu} = e^{2\Phi} \hat{g}_{\mu\nu}$  with the fiducial metric  $\hat{g}_{\mu\nu}$ . In this conformal gauge, the path integral over  $g_{\mu\nu}$  is replaced by the path integral over the Liouville field  $\Phi$  with the appropriate measure factor. Since the Einstein-Hilbert term only gives the topological contribution, we drop  $\frac{R}{2\kappa}$  term in the following.

The path integral measure of the 2D quantum gravity also contains the diffeomorphism ghost factors in the conformal gauge, but we will ignore them for our purpose since it has little to do with our interest in the geometric dynamics of the Liouville field (except for the balance of the Weyl anomaly). As for the measure factors of the Liouville field, it is expected to be ultra local and gives the kinetic term of the Liouville field. We note that the kinetic term of the Liouville action is also induced by the Weyl anomaly

$$\langle T^{\text{mat}}_{\rho}{}^{\rho} \rangle = \frac{c_{\text{matter}}}{24\pi} R, \quad (3.4)$$

of the matter action, where  $c_{\text{matter}}$  is the matter central charge. Indeed, the Liouville action may be regarded as the local Wess-Zumino like term for the Weyl anomaly.

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<sup>4</sup>More precisely, the quantization must respect the symmetry.

Collecting all these quantum contributions to the Liouville degree of freedom, the matter-gravity action can be reformulated as

$$Z \sim \int D\Phi DX e^{iS_L(\Phi) + iS_{\text{matter}}[\Phi, X]}, \quad (3.5)$$

where

$$S_L = - \int d^2x \sqrt{-\hat{g}} \left( \frac{1}{4\pi b^2} \hat{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{Q}{4\pi b} \Phi R(\hat{g}) + \frac{\Lambda_{\text{ren}}}{\kappa} e^{2\Phi} \right). \quad (3.6)$$

Here,  $\Lambda_{\text{ren}}$  is the renormalized cosmological constant. We will drop the subscript ren in the following.  $Q$  is the background charge given by  $Q = b + b^{-1}$  (see e.g. [26]) so that the total action is quantum mechanically conformally invariant. In the classical limit ( $b \rightarrow 0$ ) that we will discuss below, we have the value  $Q = b^{-1}$ .

If the matter action is conformally invariant, the Liouville field does not appear in the matter action  $S_{\text{matter}}[\Phi, X]$ . We will discuss the matter coupling in the next subsection, and we focus on the Liouville part for now. The Liouville field theory is a conformal field theory in a fixed background  $\hat{g}_{\mu\nu}$ . The path integral over  $\Phi$  is non-trivial, but we may use the trick of large number of matter fields limit again. When the number of matter fields become larger, the induced Liouville kinetic term is larger and larger,<sup>5</sup> so the quantum fluctuation of the Liouville field becomes suppressed. In (3.6),  $b^2$  becomes smaller for the larger number of matter fields, and the Planck constant becomes smaller. Therefore, although the origin of the Liouville action is purely quantum mechanical, we may treat it as if it is classical in the limit of large number of matter fields.

In analogy to the dS solution in the Einstein gravity, our interest is the dS solution of the Liouville gravity. There are two alternative viewpoints. One is to choose the background fiducial metric  $\hat{g}_{\mu\nu}$  to be dS space. Then we see that the classical equations of motion of the Liouville field becomes  $2\Lambda\kappa^{-1}e^{2\Phi} = -\frac{Q}{4\pi b}H^2$  for constant  $\Phi$ . With a convenient choice of  $\Phi = 1$ , the Hubble constant and the 2D cosmological constant (or Liouville coupling constant) is related. Note that the value of  $H$  is not that important in the physical metric  $g_{\mu\nu} = e^{2\Phi}\hat{g}_{\mu\nu}$  because it is cancelled by the factor  $\Phi$  from the Liouville equation. At this point, it is important to remind ourselves that the negative value of  $\Lambda$  corresponds to dS space in Liouville gravity (see also the discussion in the next subsection).

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<sup>5</sup>It is determined by the Weyl anomaly of the matter as  $6b^{-2} \sim -c_{\text{matter}}$ . Actually, it is negatively smaller for larger  $c_{\text{matter}}$ , and the kinetic term becomes negative. To avoid the difficulty, we may add the “non-unitary” matter with the large negative central charges. For our discussions, we always keep  $b^2$  to be positive. Note that this regime is opposite to the one studied in [25] where  $b^2$  was chosen to be negative.

The other viewpoint is to consider the Liouville equation in the flat Minkowski space with  $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$  so that it becomes  $\square\Phi = 4\pi b^2 \Lambda \kappa^{-1} e^{2\Phi}$ . The Liouville field cannot become constant, and the simplest solution is  $\Phi = -\ln(-H\eta)$ , which again gives rise to the physical dS metric  $g_{\mu\nu} = e^{2\Phi}\eta_{\mu\nu}$ . In whichever viewpoint, the matter action couples to the physical metric  $g_{\mu\nu}$ , so we may only consider the matter action in the dS space.

So far, in this section, we have treated the matter contributions as if it preserves the dS invariance. When the matter EM tensor breaks the dS invariance, the classical Liouville equation is modified and the screening effects of the Liouville coupling constant may occur. This is analogous to the matter screening of the effective cosmological constant discussed in the last section, and we will study it in the following.

### 3.2 The coupling of Liouville gravity and matter

Our main interest is to evaluate the quantum effects of gravity and matter at IR region by making use of Liouville field theory. The 2D cosmological constant has two faces, one as the coupling of the Liouville potential in terms of the Liouville action, one as the trace of the EM tensor. Let us start with the action

$$S_{L+\text{mat}}[\Phi, \phi] = - \int d^2x \sqrt{-g} \left[ \frac{1}{4\pi b^2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{Q}{4\pi b} R\Phi + \frac{\Lambda}{\kappa} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right]. \quad (3.7)$$

After taking Weyl transformation to metric,<sup>6</sup> we obtain

$$S_{L+\text{mat}}[\Phi, \phi] = - \int d^2x \sqrt{-\hat{g}} \left[ \frac{1}{4\pi b^2} \hat{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{Q}{4\pi b} \hat{R}\Phi + \frac{\Lambda}{\kappa} e^{2\Phi} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + e^{2\Phi} V(\phi) \right]. \quad (3.8)$$

$\Phi$  is the Liouville field and  $\phi$  is a matter field. The fifth term describes the interaction term between the 2D Liouville gravity and matter.

The above argument is purely classical in the Liouville degrees of freedom. As advocated before we are working in the classical Liouville regime in the large number of matter fields in mind. We will only focus on one particular degree of freedom of the matter (i.e. a scalar with  $\lambda\phi^4$  interaction), but we always assume the extra large numbers of spectator matter fields to make the classical treatment of the Liouville field theory valid.

The dS symmetry plays an important role in the determination of the trace of the EM tensor together with the conformal symmetry in Liouville field theory. However we

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<sup>6</sup>We have assumed that the Weyl anomaly cancels among Liouville part, matter part and the ghost part which we have not written down explicitly.

have seen in section 2 that if there is a massless scalar field, IR divergence will arise and break a part of the dS symmetry. The existence of a dS invariant vacuum then becomes ambiguous at least from the perturbative point of view. In this case we have additional time-dependent contributions to the effective cosmological constant.

As in the Einstein gravity case discussed in the previous section, the effective cosmological constant is given by

$$\begin{aligned}\Lambda_{\text{eff}} &= \Lambda + \kappa \langle V(\phi) \rangle \\ &= \Lambda - \frac{\kappa}{2} \langle T_{\rho}{}^{\rho} \rangle .\end{aligned}\tag{3.9}$$

Then the effective Liouville equation takes the form

$$-\frac{1}{2\pi b^2} \hat{\square} \Phi + \frac{Q}{4\pi b} \hat{R} = -2\Lambda_{\text{eff}} \kappa^{-1} e^{2\Phi} ,\tag{3.10}$$

where  $\Lambda_{\text{eff}}$  may contain the effects of the IR dS breaking from the matter contributions in (3.9). If this is the case, the Liouville field can be no longer constant with the fiducial dS metric  $\hat{g}_{\mu\nu}$ . Then the physical metric  $g_{\mu\nu}$  is not dS invariant in the semi-classical limit. In this sense, the screening of the cosmological constant gives the similar effects in the Liouville gravity to the Einstein gravity in the higher dimensions.

There is one subtle but important distinction between the Einstein gravity and the Liouville gravity that we would like to point out. In the Einstein gravity, if the energy of the universe is positive then the space-time allows the classical dS solution. This is the meaning of the positive cosmological constant in the expanding universe. However, in the Liouville gravity, the opposite is true. If the universe has the *negative* energy then the Liouville equation allows the classical dS solution (or sphere in the Euclidean signature). This difference yields an interesting consequence in the non-perturbative Liouville cosmology with meta-stable vacua [43, 44]. In our study, the sign difference makes the IR effects of the massless  $\lambda\phi^4$  theory screen rather than anti-screen the cosmological constant in  $D = 2$  in sharp contrast to the situations in  $D > 2$ .

## 4 Quantum corrections - 2D matter

By fixing the value of the Liouville field  $\Phi$  to its classical configuration, (3.8) is equal to the matter action in a fixed gravitational background. In what follows we concentrate on the dynamics of the matter field  $\phi$ . The purpose of this section is to evaluate the massless matter loop corrections to the EM tensor in the massless  $\lambda\phi^4$  theory. The loop corrections from massless matters are an interesting problem in its own right. Our main interest is the IR logarithms which are particular for massless scalar fields (and graviton) in dS space.

We are going to show explicitly that the cosmological constant indeed receives the time dependent corrections through the dS breaking expectation value of the EM tensor in the way we have discussed in section 2 and 3.

We shall work with a 2D massless minimally coupled scalar field theory with  $\lambda\phi^4$  interaction. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g} - \frac{1}{4!}\lambda\phi^4\sqrt{-g} + \Delta\mathcal{L}, \quad (4.1)$$

where  $\Delta\mathcal{L}$  consists of the counter-terms

$$\begin{aligned} \Delta\mathcal{L} = & -\frac{1}{2}\delta Z g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g} - \frac{1}{4!}\delta\lambda\phi^4\sqrt{-g} - \frac{1}{2}\delta m^2\phi^2\sqrt{-g} \\ & + \delta\xi(R - D(D-1)H^2)\phi^2\sqrt{-g} - \frac{\delta\Lambda}{8\pi G}\sqrt{-g}. \end{aligned} \quad (4.2)$$

The matter EM tensor is given by

$$\begin{aligned} T_{\mu\nu}^{\text{mat}}(x) = & (1 + \delta Z) \left( \delta^\rho_\mu \delta^\sigma_\nu - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma} \right) \partial_\rho\phi\partial_\sigma\phi - g_{\mu\nu} \left( \frac{\lambda + \delta\lambda}{4!}\phi^4 + \frac{1}{2}\delta m^2\phi^2 + \frac{\delta\Lambda}{8\pi G} \right) \\ & - 2\delta\xi \left[ g_{\mu\nu}((D-1)H^2\phi^2 + (\phi^2)^\rho{}_\rho) - (\phi^2)_{;\mu\nu} \right], \end{aligned} \quad (4.3)$$

where  $;$  denotes the covariant derivative with respect to the dS background  $g_{\mu\nu} = a(\eta)^2\eta_{\mu\nu}$ . The first term is the effect of the kinetic term and the second, third and fourth terms are the effects of the potential terms. The last term proportional to  $\delta\xi$  is the conformal counter-term.

We will calculate the vacuum expectation value (VEV) of the EM tensor by using the in-in formalism. Our calculation is regarded as a 2D analogue of that in 4D performed first in [38]. Regarding the IR logarithm,  $\ln a$ , we expect that the leading contribution to the EM tensor comes from the potential term. It is because the degree of IR divergence is weakened by derivatives: derivatives acting on the propagators reduce the number of the IR logarithms. Hence the contributions of the IR logarithms from the kinetic terms are weaker than that from the potential terms at each order of perturbative calculation. More detailed discussions including the issue of conservation of the EM tensor are given in [36]. In the following we will focus on the potential term as the leading contribution to the EM tensor and neglect the kinetic term contribution.

Our renormalization prescription in this section follows [38] in the sense that we only introduce the dS invariant counter-terms. This is motivated to keep the equations of motion of the  $\lambda\phi^4$  theory intact. As in  $D = 4$ , the dS symmetry will be broken by the renormalization. The alternative quantization to preserve the dS invariance at the sacrifice of the equations of motion will be presented in the next section.

The EM tensor deriving from the potential term is

$$T_{\mu\nu}^{\text{mat}}(x)_{\text{pot.}} = -g_{\mu\nu} \left( \frac{\lambda + \delta\lambda}{4!} \phi^4 + \frac{1}{2} \delta m^2 \phi^2 + \frac{\delta\Lambda}{8\pi G} \right) - 2\delta\xi \left[ g_{\mu\nu} ((D-1)H^2 \phi^2 + (\phi^2)_{;\rho}^{\rho}) - (\phi^2)_{;\mu\nu} \right]. \quad (4.4)$$

To evaluate its expectation value, we expand the time-evolution operator as

$$\langle \Omega | T_{\mu\nu}^{\text{mat}}(x) | \Omega \rangle \simeq \langle T \{ T_{\mu\nu}^{\text{mat}}(x) \left( 1 + i \int_C \sqrt{-g} d^2 z \mathcal{L}_{\text{int}} \right) \} \rangle, \quad (4.5)$$

with  $\mathcal{L}_{\text{int}}$  made of the order  $\lambda$  terms in (4.1), in order to take into account the first order effects in the perturbation theory. The resulting expectation value of the EM tensor includes terms of order  $\lambda^2$ . At the first order in  $\lambda$ , the expectation value of the EM tensor (4.4) can be evaluated in the free vacuum as we will see.

We begin with the evaluation of the following terms because we know they will determine the rest of (4.4).

$$\begin{aligned} & -g_{\mu\nu} \langle \Omega | \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) | \Omega \rangle \\ &= -g_{\mu\nu} \left[ \frac{\lambda}{4!} \langle \phi^4(x) \rangle + \frac{1}{2} \delta m^2 \langle \phi^2(x) \rangle \right. \\ & \quad + i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z \sqrt{-g} \langle T \{ \phi^4(x) \phi^4(z) \} \rangle - i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z' \sqrt{-g} \langle \phi^4(z') \phi^4(x) \rangle \\ & \quad + i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z \sqrt{-g} \langle T \{ \phi^2(x) \phi^4(z) \} \rangle - i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z' \sqrt{-g} \langle \phi^4(z') \phi^2(x) \rangle \\ & \quad + i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z \sqrt{-g} \langle T \{ \phi^4(x) \phi^2(z) \} \rangle - i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z' \sqrt{-g} \langle \phi^2(z') \phi^4(x) \rangle \\ & \quad + i \left( \frac{\delta m^2}{2} \right)^2 \int d^2 z \sqrt{-g} \langle T \{ \phi^2(x) \phi^2(z) \} \rangle - i \left( \frac{\delta m^2}{2} \right)^2 \int d^2 z' \sqrt{-g} \langle \phi^2(z') \phi^2(x) \rangle \\ & \quad \left. + \mathcal{O}(\lambda^3) \right]. \end{aligned} \quad (4.6)$$

We are working in the in-in formalism. The two copies of the vertices on so called  $+$  and  $-$  coordinates have been introduced:  $z$  and  $z'$  should be regarded as the vertices of the  $+$  and  $-$  types, respectively. The space-time point  $x$  at which the EM tensor operator is inserted is now assumed to be on  $+$  coordinate.

## 4.1 Order $\lambda$ potential contributions

Let us consider the first two terms in (4.6) that are order  $\lambda$  corrections to the matter EM tensor. The two diagrams corresponding to those terms are shown in Fig.1 *a1* and *a2*. The first quantity to be calculated is the mass counter-term  $\delta m^2$  which is determined



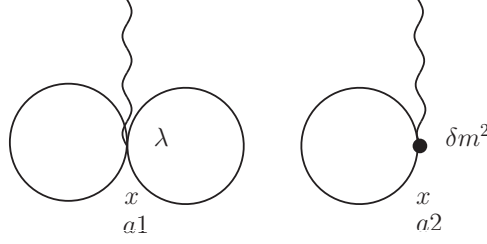


Figure 1: Order  $\lambda$  corrections to the EM tensor.

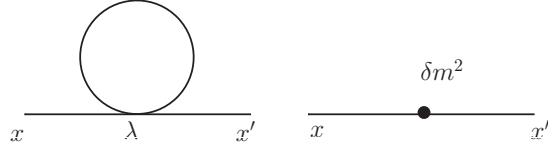


Figure 2: One-loop diagrams for mass corrections.

by the renormalization condition that the renormalized mass is zero at the initial time  $\eta_i = -1/H$  ( $t_i = 0$ ). This renormalization condition is the same as in [38, 45].

The one-loop diagrams shown in Fig. 2 give the one-loop scalar squared mass  $M_{1\text{-loop}}^2$ .

$$-iM_{1\text{-loop}}^2(x, x') = -i \left[ \frac{\lambda}{2} \Delta(x, x) + \delta m^2 \right] \delta^2(x - x'). \quad (4.7)$$

Substitution of (2.18) into (4.7) yields

$$M_{1\text{-loop}}^2(x, x') = \left[ \frac{\lambda}{2} \alpha \beta \{ 2 \ln(a(\eta)) + \frac{2}{\omega} + A' \} + \delta m^2 \right] \delta^2(x - x'), \quad (4.8)$$

where  $A' = C - \gamma$ . The renormalization condition for the mass mentioned above reads

$$\delta m^2 = -\frac{\lambda}{2} \alpha \beta \left( \frac{2}{\omega} + A' \right) + O(\lambda^2). \quad (4.9)$$

It follows that

$$M_{1\text{-loop}}^2(x, x') = \frac{\lambda}{4\pi} \ln(a(\eta)) \delta^2(x - x'). \quad (4.10)$$

To eliminate UV divergence from the EM tensor completely, we need to introduce the counter-term for the cosmological constant  $\delta\Lambda$  in addition to the mass counter-term. The diagrams in Fig.1 a1 and a2 with the mass counter-term (4.9) give

$$\begin{aligned} \frac{\lambda}{4!} \langle \phi^4(x) \rangle + \frac{1}{2} \delta m^2 \langle \phi^2(x) \rangle &= \frac{\lambda}{4!} \cdot 3 \Delta_{++}^2(x, x) + \frac{1}{2} \delta m^2 \Delta_{++}(x, x) \\ &= \frac{\lambda}{2} \left( \frac{1}{4\pi} \left( \frac{H^2}{4\pi} \right)^{\frac{\omega}{2}} \frac{\Gamma(1-\omega)}{\Gamma(1-\omega/2)} \right)^2 \\ &\quad \times \left( -\frac{1}{\omega^2} - \frac{1}{\omega} A' + \ln^2 a(\eta) - \frac{A'^2}{4} \right). \end{aligned} \quad (4.11)$$

Accordingly,  $\delta\Lambda$  is determined by the requirement that

$$-g_{\mu\nu} \left[ \frac{\lambda}{4!} \langle \phi^4(x) \rangle + \frac{1}{2} \delta m^2 \langle \phi^2(x) \rangle \right] - \frac{\delta\Lambda}{8\pi G} g_{\mu\nu} = (\text{finite}). \quad (4.12)$$

Thus we have

$$\frac{\delta\Lambda}{8\pi G} = \frac{\lambda}{32\pi^2} \left( \frac{H^2}{4\pi} \right)^{-\omega} \frac{\Gamma^2(1-\omega)}{\Gamma^2(1-\omega/2)} \left[ \frac{1}{\omega^2} + \frac{1}{\omega} A' \right] + \frac{\delta\Lambda_{\text{fin}}}{8\pi G}, \quad (4.13)$$

where  $\delta\Lambda_{\text{fin}}$  is the finite part of the counter-term for which we shall choose  $\delta\Lambda_{\text{fin}} = A'^2/4$  at this stage. As a result, the EM tensor at order  $\lambda$  is obtained as

$$\langle T_{\mu\nu}^{\text{mat}} \rangle_{\text{pot. } \mathcal{O}(\lambda)} = -g_{\mu\nu} \frac{\lambda}{32\pi^2} \ln^2 a(\eta). \quad (4.14)$$

As we have mentioned in the last subsection, the order  $\lambda$  contribution corresponds to the zeroth order result in the perturbative expansion. In the next subsection and Appendix A, we consider the order  $\lambda^2$  contribution in order to include the effect from the interaction vertices.

## 4.2 Order $\lambda^2$ potential contributions

In order to evaluate the renormalization of the cosmological constant at the  $\lambda^2$  order, we must deal with the three-loop diagrams. We note that the counter-terms for the coupling constant  $\delta\lambda$  and the conformal coupling  $\delta\xi$  are absent in our computation. It is because the one-loop correction to the  $\lambda\phi^4$  interaction term is not UV divergent in 2D and we do not have the terms proportional to the mixing of UV and IR divergent term  $\omega^{-1} \cdot \ln a(\eta)$  that are supposed to be canceled by  $\delta\xi$  [38]. The detail of calculation of the three loop diagrams is presented in Appendix A. The leading contribution to the EM tensor at this order is

$$\langle T_{\mu\nu}^{\text{mat}} \rangle_{\text{pot. } \mathcal{O}(\lambda^2)} \sim -g_{\mu\nu} \frac{1}{8\pi} \frac{\lambda^2}{(4\pi)^2 H^2} \ln^4 a(\eta). \quad (4.15)$$

The dimensionless expansion parameter can be regarded as  $\lambda/H^2$  and our perturbative computation is valid as long as  $\ln a(\eta) < H\lambda^{-1/2}$ .

From (4.14) and (4.15), we finally obtain the effective cosmological constant at the order  $\lambda^2$ ,

$$\begin{aligned} \Lambda_{\text{eff}} &= \Lambda - \frac{\kappa}{2} \langle T^{\text{mat} \rho}_{\rho} \rangle \\ &\sim \Lambda + (\text{Weyl anomaly}) + \frac{\kappa\lambda}{32\pi^2} \ln^2 a(\eta) + \frac{1}{8\pi} \frac{\kappa\lambda^2}{(4\pi)^2 H^2} \ln^4 a(\eta), \end{aligned} \quad (4.16)$$

where the Weyl anomaly is given by  $\langle T^{\text{mat}\rho}_{\rho} \rangle = R/(24\pi)$  [46]. The effective cosmological constant has time dependence as expected, and it increases as time passes. We recall that in the Liouville gravity the dS vacuum corresponds to negative value of  $\Lambda$ . Therefore, the cosmological constant evolves from the negative value toward zero, leading to the Minkowski space (within our approximation). It means that the massless  $\lambda\phi^4$  theory in  $D = 2$  shows the IR screening effect on the cosmological constant. As we noted in section 3, it crucially relies on the nature of the classical Liouville gravity. For comparison, see for instance [36] where massless  $\lambda\phi^4$  theory in 4D dS space has been investigated and the effective cosmological constant shows the anti-screening effect at the perturbative level.

## 5 A possible dS invariant $\lambda\phi^4$ theory with infrared counter-terms

As we mentioned in the introduction, the conclusion that the  $\lambda\phi^4$  theory screens the cosmological constant in late time is puzzling in  $D = 2$  dimensions. In flat Minkowski space, the IR limit of  $\lambda\phi^4$  theory in  $D = 2$  dimensions is equivalent to a free Majorana fermion from the Landau-Ginzburg construction [33, 34]. The free Majorana fermion is conformally invariant and does not show any IR pathology in dS space.

The similar question was addressed in the equivalence between Sine-Gordon model and massive Thirring model. In [35], it was claimed that the equivalence is lost in dS space due to the quantum IR dS breaking effects of the massless propagator used in the perturbative construction of the Sine-Gordon model. However, as we presently show the breaking effects are local and, if we allow the *dS non-invariant* local counter-terms, the dS breaking effects can be completely removed. Then the equivalence between Sine-Gordon model and the massive Thirring model holds in dS space. We plan to study in detail this equivalence in another work.

With this viewpoint, let us reconsider the  $\lambda\phi^4$  theory in dS space

$$S = \int d\eta dx \left( \frac{1}{2} (\partial_{\eta}\phi\partial_{\eta}\phi - \partial_x\phi\partial_x\phi) - (H\eta)^{-2} \frac{\lambda}{4!} \phi^4 \right). \quad (5.1)$$

In perturbation theory, if we use the propagator with the IR regularization given by

$$\langle \phi(\eta_1, x_1) \phi(\eta_2, x_2) \rangle = -\frac{1}{4\pi} \ln \left( \frac{-(\eta_1 - \eta_2)^2 + (x_1 - x_2)^2}{H^{-2}} \right), \quad (5.2)$$

the dS invariance is broken. This propagator preserves the Poincaré invariance in flat Minkowski space (while breaking the scale invariance), but it breaks the dS invariance. In particular, the dS isometry  $\eta \rightarrow b\eta$  and  $x \rightarrow bx$  is broken. To obtain the dS invariance,

the expression must be written by using the dS invariant length

$$y = \frac{-(\eta_1 - \eta_2)^2 + (x_1 - x_2)^2}{\eta_1 \eta_2}, \quad (5.3)$$

but then the simple replacement in the propagator such as

$$\langle \phi(\eta_1, x_1) \phi(\eta_2, x_2) \rangle = -\frac{1}{4\pi} \ln \left( \frac{-(\eta_1 - \eta_2)^2 + (x_1 - x_2)^2}{\eta_1 \eta_2} \right). \quad (5.4)$$

does not solve the massless equations of motion  $\square\phi = 0$  (except at  $\eta = -\infty$ ).<sup>7</sup> This is the origin of the dS breaking effect in massless (interacting) scalar field.

We would like to see if there is any counter-term to cancel this dS symmetry breaking from the IR effect. For this purpose, let us first note that the modified propagator (5.4), which is dS invariant, may be obtained by adding the curvature term to the action:<sup>8</sup>

$$\begin{aligned} \tilde{S} &= \int d\eta dx \left( \frac{1}{2} \left( \partial_\eta \tilde{\phi} \partial_\eta \tilde{\phi} - \partial_x \tilde{\phi} \partial_x \tilde{\phi} + \frac{2}{\eta^2} \tilde{\phi} \right) - (H\eta)^{-2} \frac{\lambda}{4!} \tilde{\phi}^4 \right) \\ &= \int d\eta dx \left( \frac{1}{2} \left( \partial_\eta \tilde{\phi} \partial_\eta \tilde{\phi} - \partial_x \tilde{\phi} \partial_x \tilde{\phi} + 2(\square \ln \eta) \tilde{\phi} \right) - (H\eta)^{-2} \frac{\lambda}{4!} \tilde{\phi}^4 \right). \end{aligned} \quad (5.5)$$

This action possesses dS invariance at quantum as well as classical level, in contrast with the original action (4.1) in the sense that the action (5.5) generates the dS invariant Feynman diagrams (as long as we use (5.4) as the propagator for  $\tilde{\phi}$ ).

Now let us go back to the original action with the dS breaking propagator. The model can be conveniently expressed by introducing  $\phi = \tilde{\phi} - \ln \eta$  in (5.5). Note that this field redefinition does not preserve the manifest dS invariance. The resulting action has the same kinetic term for the  $\lambda\phi^4$  theory with which we started, but we have extra (seemingly) dS breaking terms, which we regard as the counter-term to the original  $\lambda\phi^4$  action:

$$\tilde{S} = \int d\eta dx \left( \frac{1}{2} (\partial_\eta \phi \partial_\eta \phi - \partial_x \phi \partial_x \phi) - (H\eta)^{-2} \frac{\lambda}{4!} (\phi + \ln \eta)^4 \right) \quad (5.6)$$

up to “cosmological constant”  $\eta^{-2}$  that is independent of the field. If we evaluate Feynman diagrams in perturbation theory for (5.6) (with respect to  $\lambda$ ), the counter-terms such as  $\phi^3 \ln \eta$ ,  $\phi^2 (\ln \eta)^2$  will cancel the dS breaking IR regularization in the propagator (5.2) for  $\phi$ . Note that in order to fully recover the dS invariance, we have to renormalize the operator as well. For instance, the correlation functions of  $\phi$  itself is not dS invariant (even

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<sup>7</sup>It rather solves the Liouville equation without cosmological constant  $\square\phi(\eta, x) + R(\eta, x) = 0$ . It is related to the fact that the Euclidean dS space is a sphere, and a sphere does not allow the massless propagator.

<sup>8</sup>The kinetic term is the same as that for the Liouville field theory, which does not break the dS invariance.

if we added the counter-term (5.6)), but we have to consider the renormalized operator  $\tilde{\phi} = \phi + \ln \eta$ . This also applies to the EM tensor computation of the previous sections.

It may be a matter of taste if the theory with the IR counter-term (5.6) should be called as the “dS invariant  $\lambda\phi^4$  theory”. We should also recall that this is not the only way to recover the dS invariance anyway. Note that our concern was the dS breaking IR effects of the massless propagator, and it is trivially possible to avoid the dS breaking just by adding a tiny mass term from the beginning. If we, however, treat the “mass term” as a perturbation around massless theory, the same mechanism of the IR counter-terms should have worked. The dS breaking effects of the massless propagator is precisely cancelled by the “dS breaking” mass perturbation so that the summed diagram recovers the expected dS invariance of the massive scalar propagator.

## 6 Discussion

In this paper we have constructed a 2D model of quantum gravity coupled to matter in dS space to explore the IR quantum effects in lower-dimensional dS space. The model (3.8) is described by the Liouville field theory coupled to matter which is minimally interacting with the Liouville field  $\Phi$  through the physical metric. Once the fiducial metric is taken to be dS space, the classical Liouville field equation has a constant solution, and in this case the model reduces to an ordinary matter theory in the fixed dS background. One eminent feature of our model is that the cosmological constant in 2D dS space has the negative sign that follows from the Liouville field equation. This property of cosmological constant is opposite to the case of the Einstein gravity in  $D > 2$ .

As a concrete matter Lagrangian, we have studied a massless scalar field theory with  $\lambda\phi^4$  interaction minimally coupled to Liouville gravity. In dS space, the massless scalar propagator contains the IR divergence in the long wavelength limit and the IR logarithm appears due to the cutoff regularization of the IR divergence. Based on the in-in formalism, we have computed the VEV of the EM tensor of order  $\lambda^2$ . The resulting VEV (4.15) has a time dependence through the IR logarithms, and as a consequence, the effective cosmological constant shows the screening effect at late time such that the absolute value decreases with time. This should be in contrast with the situations in  $D > 2$ , in which the cosmological constant is anti-screened in the  $\lambda\phi^4$  theory.

The degree of IR divergence in 2D, however, has turned out to be the same as that in 4D [38]. If it were in Minkowski space, the degree of IR divergence in 2D would have been stronger than that in 4D. Nevertheless, the propagator in dS space is more complicated, and the structure varies by dimension. We do see the IR logarithms  $\ln a$  both in 2D and

4D dS space, but we do not observe the enhanced degree of IR divergence in the VEV of the energy-momentum tensor. Based on this observation, we may expect that the same argument for the power-counting of the leading term of the IR logarithm in 4D dS space applies to our 2D case as well. According to [38, 36], at  $L$ -loop order, the VEV of the energy-momentum tensor scales as:

$$\langle T_{\mu\nu}^{\text{mat}} \rangle \sim -g_{\mu\nu} \left( \frac{1}{H^2} \right)^{L-2} (\lambda \ln^2 a)^{L-1},$$

where the  $L$  dependence of the power law of the Hubble constant compensates the mass dimension coming from the dimensionful coupling constant  $\lambda$ . Then we may apply the known methods to resum the leading IR logarithms [47, 48] in our  $D = 2$  case, but we leave the detailed study for a separate work.

Once the effective cosmological constant is time dependent due to the matter quantum effects, the classical Liouville field dynamics will be affected as in the case of 4D Einstein gravity. The matter dynamics modifies the classical Liouville equation through the time-dependent matter EM tensor and it hinders for the physical metric to possess the dS solution. We would like to investigate the dynamics of the subsequent Liouville field and its quantum effects in the back-reacted solution in a future work.

In this paper, on the other hand, we have addressed an alternative possibility to add the dS non-invariant counter-terms in order to keep the dS invariance in correlation functions. As a concrete model, we have reconsidered the same massless  $\lambda\phi^4$  theory. Naive truncation of the dS breaking term from the propagator alters the equation of motion of the free massless scalar field and the modified propagator no longer satisfies the original equation of motion.<sup>9</sup> However, we have found that we are able to cancel the effects of the IR logarithms in correlation functions by adding the dS breaking time-dependent counter-terms which come from the dS non-invariant interaction terms. If all dS breaking terms can be cancelled in this manner, we may find the dS breaking effects neither in the EM tensor nor in any other observables in dS space. It would be interesting to see if the same mechanism applies to the  $D = 4$  theories.

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<sup>9</sup>In [18], it was claimed that the modified propagator *can* correspond to the usual scale-invariant power spectrum, rather than original dS breaking propagator does.

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## A Order $\lambda^2$ corrections to the energy-momentum tensor

In this appendix we outline the calculation of order  $\lambda^2$  loop corrections to the EM tensor. From (4.6), we start with

$$\begin{aligned}
& -g_{\mu\nu} \langle \Omega | \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) | \Omega \rangle_{\mathcal{O}(\lambda^2)} \\
&= -g_{\mu\nu} \left[ i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z \sqrt{-g(z)} \langle T \{ \phi^4(x) \phi^4(z) \} \rangle - i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z' \sqrt{-g(z')} \langle \phi^4(z') \phi^4(x) \rangle \right. \\
&\quad + i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z \sqrt{-g} \langle T \{ \phi^2(x) \phi^4(z) \} \rangle - i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z' \sqrt{-g} \langle \phi^4(z') \phi^2(x) \rangle \\
&\quad + i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z \sqrt{-g} \langle T \{ \phi^4(x) \phi^2(z) \} \rangle - i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z' \sqrt{-g} \langle \phi^2(z') \phi^4(x) \rangle \\
&\quad \left. + i \left( \frac{\delta m^2}{2} \right)^2 \int d^2 z \sqrt{-g} \langle T \{ \phi^2(x) \phi^2(z) \} \rangle - i \left( \frac{\delta m^2}{2} \right)^2 \int d^2 z' \sqrt{-g} \langle \phi^2(z') \phi^2(x) \rangle \right]. \tag{A.1}
\end{aligned}$$

Note that we chose  $x$  and  $z$  as  $+$  type vertices and  $z'$  as  $-$  type vertex as explained in section 4. The Feynman diagrams are shown in Fig.3 where the second line of (A.1) corresponds to  $b1 - b4$ , the third line corresponds to  $c1$  and  $c2$ , the fourth line corresponds to  $d1$  and  $d2$ , and the last line corresponds to  $e1$  and  $e2$ . Wick contractions and a simple calculation by the use of (2.17) give

$$\begin{aligned}
& \langle \Omega | \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) | \Omega \rangle_{\mathcal{O}(\lambda^2)} \\
&= i \int d^2 z \sqrt{-g} \left[ \frac{\lambda^2}{24} (i \Delta_{++}^4(x, z) - i \Delta_{+-}^4(x, z)) \right. \\
&\quad \left. + \frac{1}{2} \left( \frac{\lambda}{2} i \Delta(x, x) + \delta m^2 \right) \left( \frac{\lambda}{2} i \Delta(z, z) + \delta m^2 \right) (i \Delta_{++}^2(x, z) - i \Delta_{+-}^2(x, z)) \right] \\
&\equiv I_1 + I_2, \tag{A.2}
\end{aligned}$$

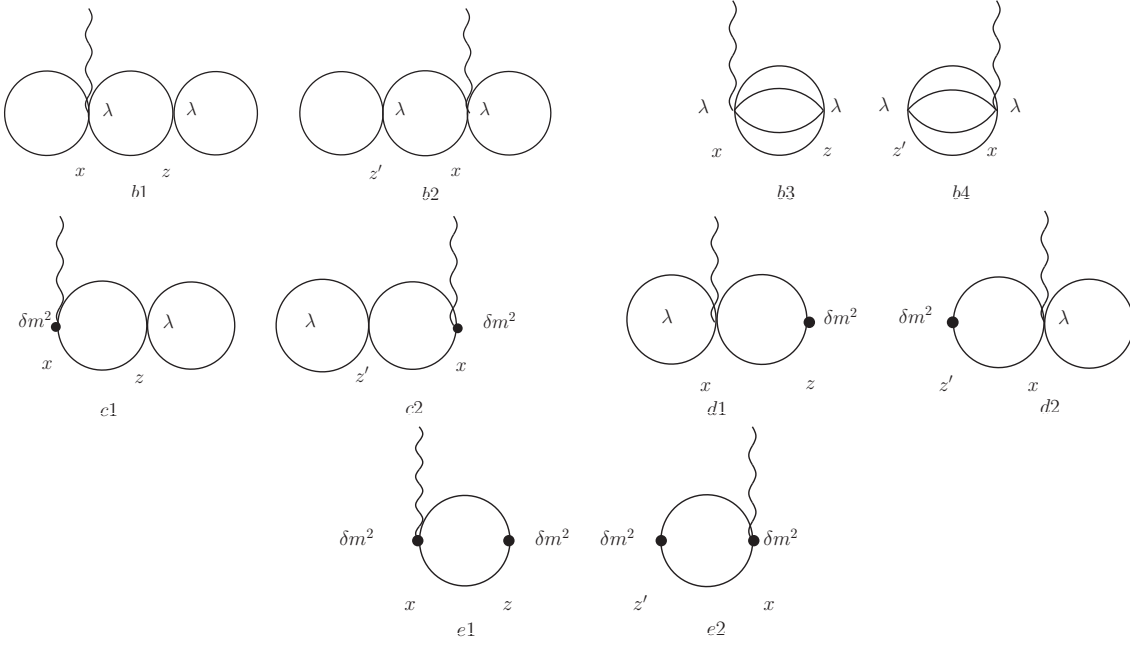


Figure 3: Order  $\lambda^2$  corrections to the EM tensor  $T_{\mu\nu}^{\text{pot}}$ .

where we have collected the integrations into that of  $z$  since  $z$  and  $z'$  are dummy variables, and defined  $I_1$  and  $I_2$  by

$$I_1(\eta) \equiv i \int d^2 z \sqrt{-g} \left[ \frac{\lambda^2}{24} (i\Delta_{++}^4(x, z) - i\Delta_{+-}^4(x, z)) \right], \quad (\text{A.3})$$

$$I_2(\eta) \equiv i \int d^2 z \sqrt{-g} \left[ \frac{1}{2} \left( \frac{\lambda}{2} i\Delta(x, x) + \delta m^2 \right) \left( \frac{\lambda}{2} i\Delta(z, z) + \delta m^2 \right) (i\Delta_{++}^2(x, z) - i\Delta_{+-}^2(x, z)) \right]. \quad (\text{A.4})$$

Let us first consider the  $I_1(\eta)$ . The integrand is expanded as

$$\begin{aligned} & i\Delta_{++}^4(x, z) - i\Delta_{+-}^4(x, z) \\ &= \{i\Delta_{++}^2(x, z) + i\Delta_{+-}^2(x, z)\} \{i\Delta_{++}(x, z) + i\Delta_{+-}(x, z)\} \{i\Delta_{++}(x, z) - i\Delta_{+-}(x, z)\} \\ &= \alpha^4 \{ \gamma^4(y_{++}) - \gamma^4(y_{+-}) + 4\beta \ln(aa_z)(\gamma^3(y_{++}) - \gamma^3(y_{+-})) \\ & \quad + 6\beta^2 \ln^2(aa_z)(\gamma^2(y_{++}) - \gamma^2(y_{+-})) + 4\beta^3 \ln^3(aa_z)(\gamma(y_{++}) - \gamma(y_{+-})) \}, \end{aligned} \quad (\text{A.5})$$

with abbreviations  $y_{++} = y_{++}(x, z)$ ,  $y_{+-} = y_{+-}(x, z)$ ,  $a = a(\eta)$  and  $a_z = a(\eta_z)$ . It is clear from (2.12) that we can take a limit  $\omega \rightarrow 0$  safely in (A.5) since the terms that include  $\omega$  in their denominators offset each other. Thus we have

$$\begin{aligned} & i\Delta_{++}^4(x, z) - i\Delta_{+-}^4(x, z) \\ &= \left( \frac{1}{4\pi} \right)^4 \left[ \ln^4 \left( \frac{y_{++}}{4} \right) - \ln^4 \left( \frac{y_{+-}}{4} \right) - (4C + 4 \ln(aa_z)) \left\{ \ln^3 \left( \frac{y_{++}}{4} \right) - \ln^3 \left( \frac{y_{+-}}{4} \right) \right\} \right. \\ & \quad \left. + (6C^2 - 2\pi^2 - 36C \ln(aa_z) + 6 \ln^2(aa_z)) \left\{ \ln \left( \frac{y_{++}}{4} \right) - \ln \left( \frac{y_{+-}}{4} \right) \right\} \right]. \end{aligned} \quad (\text{A.6})$$



Next we integrate (A.6) noting  $\sqrt{-g} = a_z^2$  and

$$\begin{aligned} y_{++}(x, z) &= aa_z H^2(r^2 - \Delta\eta^2 + 2ie|\Delta\eta|), \\ y_{+-}(x, z) &= aa_z H^2(r^2 - \Delta\eta^2 - 2ie\Delta\eta), \end{aligned} \quad (\text{A.7})$$

with  $r = |\vec{x} - \vec{z}|$  and  $\Delta\eta = \eta - \eta_z$ . The cut prescription allows us to write  $\ln y$  as [38, 36]

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \ln y_{++} &= \ln [aa_z H^2(\Delta\eta^2 - r^2)] + i\pi\theta(\Delta\eta^2 - r^2), \\ \lim_{\epsilon \rightarrow 0} \ln y_{+-} &= \ln [aa_z H^2(\Delta\eta^2 - r^2)] - i\pi\theta(\Delta\eta^2 - r^2)(\theta(\Delta\eta) - \theta(-\Delta\eta)), \end{aligned} \quad (\text{A.8})$$

and then the interval of integration becomes

$$\int d^2z = \int_{-\frac{1}{H}}^{\eta} d\eta_z \int_0^{\Delta\eta} dr. \quad (\text{A.9})$$

It means that the contribution from outside of the past light cone vanishes due to  $i\Delta_{++} = i\Delta_{+-}$  for either  $r^2 > \Delta\eta^2$  or  $\Delta\eta < 0$ . Using (A.8) and (A.9), we have

$$\begin{aligned} I_1(\eta) &= i \frac{\lambda^2}{24(4\pi)^4} \int_{-\frac{1}{H}}^{\eta} d\eta_z a_z^2 \Delta\eta \left[ 8\pi i \ln^3(aa_z H^2 \Delta\eta^2 / 4) \right. \\ &\quad + \{8\pi i K_1 - 6\pi i(4C + 4 \ln(aa_z))\} \ln^2(aa_z H^2 \Delta\eta^2 / 4) \\ &\quad + \{8\pi i(K_2 - \pi^2) - 6\pi i K_5(4C + 4 \ln(aa_z)) \\ &\quad + 4\pi i(6C^2 - 2\pi^2 - 36C \ln(aa_z) + 6 \ln^2(aa_z))\} \ln(aa_z H^2 \Delta\eta^2 / 4) \\ &\quad + 8\pi i(K_3 - \pi^2 K_4) - (6\pi i K_6 - 2\pi^3 i)(4C + 4 \ln(aa_z)) \\ &\quad + 4\pi i K_4 \{6C^2 - 2\pi^2 - 36C \ln(aa_z) + 6 \ln^2(aa_z)\} \\ &\quad \left. + 2\pi i \{-4C^3 - 12 \ln(aa_z) - 12C \ln^2(aa_z) - 4 \ln^3(aa_z)\} \right], \end{aligned} \quad (\text{A.10})$$

where  $K_n$  ( $n = 1, \dots, 6$ ) are some constants which are not important in the subsequent discussions. For the time integral, it is convenient to change the variable  $\eta_z$  to  $a_z$ ,

$$\int_{-\frac{1}{H}}^{\eta} d\eta_z = \int_1^a \frac{da_z}{H a_z}. \quad (\text{A.11})$$

The result of integral (A.10) has a very long expression and we shall avoid to present the full expression here because we are interested in the late time (namely  $\eta \ll -H^{-1}$ ) behavior of  $I_1(\eta)$  (and  $I_2(\eta)$ ). The leading contributions from  $I_1$  at that time can be extracted by

$$I_1(\eta) \sim \frac{\lambda^2}{24\pi(4\pi)^2 H^2} \ln^4 a. \quad (\text{A.12})$$

Next we move on to the evaluation of the  $I_2(\eta)$ . It can be done in a way similar to that applied for the  $I_1(\eta)$  and gives a simple expression.

$$\begin{aligned}
I_2(\eta) &= \frac{i}{2} \frac{\lambda^2}{(4\pi)^4} \ln a \int d^2 z \ln a_z \\
&\quad \times \left\{ \ln^2 y_{++} - \ln^2 y_{+-} - 2(\ln(aa_z) + 2\ln 2 + C)(\ln y_{++} - \ln y_{+-}) \right\} \\
&= -\frac{2\pi\lambda^2}{(4\pi)^4 H^2} \ln a \int_1^a da_z \ln a_z \left( -2 - C + 2\ln(a_z^{-1} - a^{-1}) \right) (a_z^{-1} - a^{-1}). \quad (\text{A.13})
\end{aligned}$$

The integral of  $a_z$  gives the result for  $I_2(\eta)$ ,

$$\begin{aligned}
I_2(\eta) &= -\frac{2\pi\lambda^2}{(4\pi)^4 H^2} \left[ -\frac{2}{3} \ln^4 a - \frac{C}{2} \ln^3 a - \left( \frac{\pi^2}{3} - C \right) \ln^2 a \right. \\
&\quad \left. - 2\text{Li}_2(a^{-1}) - \text{Li}_3(a^{-1}) + \frac{2+C}{a} + \frac{\pi^2}{3} + 2\text{Li}_3(1) - 2 - C \right], \quad (\text{A.14})
\end{aligned}$$

where  $\text{Li}_n(x)$  denotes the polylogarithm function which decays for small  $x$ . In this case, the leading contributions to the EM tensor is given by

$$I_2(\eta) \sim \frac{\lambda^2}{12\pi(4\pi)^2 H^2} \ln^4 a. \quad (\text{A.15})$$

Then the total contribution is given by (A.12) and (A.15),

$$I_1 + I_2 \sim \frac{1}{8\pi} \frac{\lambda^2}{(4\pi)^2 H^2} \ln^4 a. \quad (\text{A.16})$$

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